U-7924

AECD - 2067

UNITED STATES ATOMIC ENERGY COMMISSION

MOTION OF AN ION IN AN APPROXIMATELY UNIFORM MAGNETIC FIELD

by

R. R. Dempster

University of California
Radiation Laboratory

Date of Manuscript: December 1, 1943
Date Declassified: June 23, 1948

Technical Information Branch, Oak Ridge, Tennessee
AEC, Oak Ridge, Tenn., 2-23-49--850-A1466

Printed in U.S.A.
PRICE 5 CENTS

copy #90

17438

MOTION OF AN ION IN AN APPROXIMATELY UNIFORM MAGNETIC FIELD

By R. R. Dempster

MOTION IN A UNIFORM FIELD

Consider a particle of mass m and charge e moving with speed v in a plane perpendicular to the lines of force of a uniform magnetic field of intensity H. If e is in esu, one changes to emu by dividing c, where c is the velocity of light. The force on the moving charge due to the magnetic field is known to be everywhere at right angles to the magnetic field and to the velocity vector, hence, it lies in the plane of motion and is proportional to e, v, and H, to wit:

$$f = \frac{e}{c} vH. \tag{1}$$

This force, being always normal to the velocity vector, cannot change the magnitude of that vector, but only its direction, so it is a centripetal force, and one can write

$$\frac{mv^2}{R} = \frac{e}{c} vH, \tag{2}$$

whence

$$R = \frac{cmv}{eH} \tag{3}$$

The motion is a simple circular motion, with angular velocity

$$\text{omega} = \frac{v}{R} = \frac{eH}{mc}. \tag{4}$$

SLIGHTLY NON-UNIFORM FIELD

At every instant, the ion describes an arc of a circle of the radius given by equation 3, which is the circle of curvature for the point through which the ion is passing. If in a field of intensity H_0 the radius is R_0,

$$R_0 = \frac{cmv}{eH_0}. \tag{5}$$

Dividing equation 4 by equation 5,

$$\frac{R}{R_0} = \frac{H_0}{H}. \tag{6}$$

If the unit of length is taken as R_0, and if the field strength H is expressed in terms of H_0:

$$H = H_0 (1 + h), \tag{7}$$

h being the fraction (positive or negative) by which H differs from H_0, then

$$R = \frac{1}{1+h}. \tag{8}$$

In order to determine the effect of a small variation in the field strength over a short element of path, we consider an imaginary problem in which the field is 1 (i.e, H_0) everywhere along the path, except along the short element under consideration, where it is $1+h$. This situation is indicated in Figure 1, where the field is unity except between A and B. Supposing $h < 0$, one finds that an ion, which in a uniform field would describe an arc OABC of radius $R = 1$, actually describes the arc OAED, with radii of 1 along OA and ED, but with the greater radius $1/(1+h)$ from A to E. Now, if beyond E the ion travels in a circle of radius 1, the only essential change introduced into its path is that its direction is different at E than it would have been at B (in the uniform-field case). Thus, the change in the orbit produced by the nonuniformity from A to B merely rotates the whole orbit about A through an angle like BAE (Figure 2). The difference in direction ψ may be calculated as follows: Along AB the direction changes by Δt, where $\Delta t = \frac{\widehat{AB}}{1}$. Along AE the direction changes by $\Delta t' = \frac{\widehat{AE}}{1/(1+h)} =$ AE $(1+h)$. Now, if AB and AE are arcs corresponding to equal intervals of time, —v being constant regardless of the field strength—then AB = AE, so that

$$\Delta t' = \Delta t\,(1 + h). \tag{9}$$

The angle ψ is then the difference between $\Delta t'$ and Δt, so that

$$\psi = -h\Delta t, \tag{10}$$

where, since $h<0$, the negative sign is necessary to make ψ positive.

We now consider the effect of the field inhomogeneity along AB upon other points on the orbit (Figure 3). Because of the rotation of the orbit about A through the angle ψ, a point such as P will be displaced to Q, where $\overline{PQ} = D\psi$. The x and y components of $\overline{PQ}$ will be

$$\left.\begin{aligned} \text{delta } x &= -\,\overline{PQ}\sin\phi, \\ \text{delta } y &= \overline{PQ}\cos\phi: \end{aligned}\right\} \tag{11}$$

Now, as the radii of the unperturbed orbits are unity,

$$\left.\begin{aligned} LP &= \sin s - \sin t, \\ AL &= \cos t - \cos s, \end{aligned}\right\} \tag{12}$$

whence

$$\left.\begin{aligned} \sin\phi &= \frac{\sin s - \sin t}{D}, \\ \cos\phi &= \frac{\cos t - \cos s}{D}. \end{aligned}\right\} \tag{13}$$

Therefore,

$$\left.\begin{aligned} \text{delta } x &= -\frac{\cancel{D}\,\psi(\sin s - \sin t)}{\cancel{D}}, \\ \text{delta } y &= \frac{\cancel{D}\,\psi(\cos t - \cos s)}{\cancel{D}}. \end{aligned}\right\} \tag{14}$$

Substituting for ψ from equation 10 in equation 14 we find

$$\left.\begin{aligned} \text{delta } x &= h\,(\sin s - \sin t)\ \Delta t, \\ \text{delta } y &= h\,(\cos s - \cos t)\ \Delta t. \end{aligned}\right\} \tag{15}$$

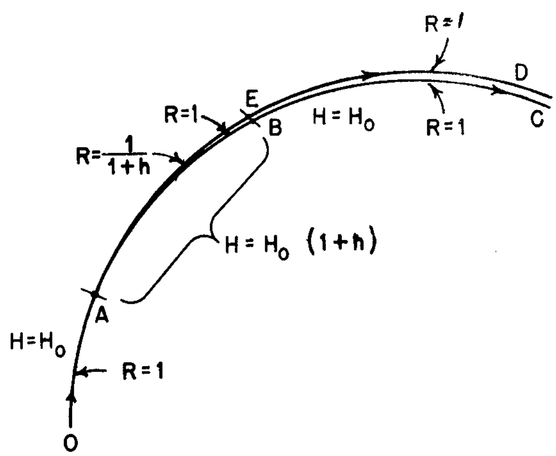

Figure 1.

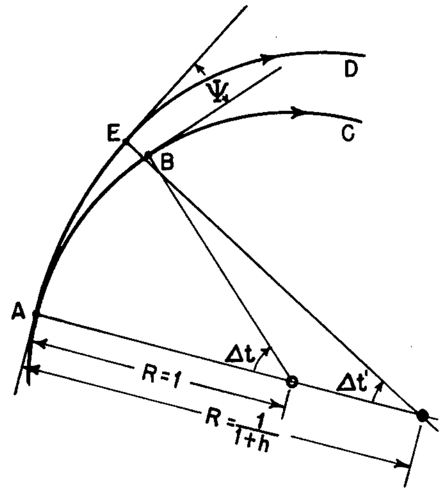

Figure 2.

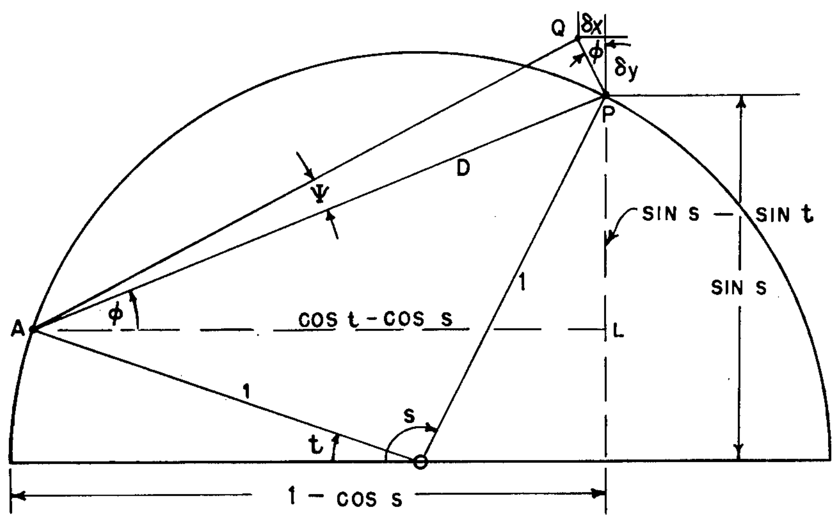

Figure 3.

This shows the effect of a small inhomogeneity in the field for an element of path Δt. To get the effect of a continuous variation in field, one integrates expressions 15 along the orbit, for all values of t up to s:

$$\left.\begin{aligned} x_{1\alpha} &= \int_0^s h_\alpha(t)\,(\sin s - \sin t)\,dt, \\ y_{1\alpha} &= \int_0^s h_\alpha(t)\,(\cos s - \cos t)\,dt. \end{aligned}\right\} \qquad (16)$$

Equations 16 give the corrections to be applied to any point on the orbit, thus giving the deviations of the coordinates from those on the simple circular orbit which would be obtained in a uniform field.

The coordinates of P are:

$$\left.\begin{aligned} x_{c\alpha} &= 1 - \cos s, \\ x_{c\alpha} &= \sin s, \end{aligned}\right\} \qquad (17)$$

so the coordinates of Q are

$$\begin{aligned} x_\alpha &= x_{c\alpha} + x_{1\alpha} = 1 - \cos s + \int_0^s h_\alpha(t)\,(\sin s - \sin t)\,dt, \\ y_\alpha &= y_{c\alpha} + y_{1\alpha} = \sin s + \int_0^s h_\alpha(t)\,(\cos s - \cos t)\,dt. \end{aligned} \qquad (18)$$

Strictly speaking, the values of $h_\alpha(t)$ must be taken along the actual orbit, and not along the unperturbed circle. However, if the field varies slowly enough from point to point, the use of h_α values determined along the unperturbed orbit will give sufficient accuracy.

END OF DOCUMENT